ISSN 2229-5518

Stability analysis of finite difference schemes for heat equation with various thermal conductivity

M. Rozina Khatun & Md. Shajib Ali

Abstract- This paper presents the stability analysis of one-dimensional heat equation. We study the analytical solution of heat equation as an initial value problem in infinite space and realize the qualitative behavior of the solution in terms of heat diffusion co-efficient. We obtain the numerical solution of this equation by using the first order explicit centered difference scheme (forward time and central space (FTCS Techniques)) and a second order Crank-Nicolson scheme (CNS techniques) for prescribed initial and boundary data. We implement the numerical scheme by developing a computer programming code and present the stability analysis of explicit centered difference schemes and Crank-Nicolson scheme for heat equation. It's found that CNS schemes gives better pointwise solutions then ECDS in terms of time step selection.

Keyword- Heat Equation, Finite Difference Method, Explicit Scheme, Crank-Nicolson Scheme, Stability Condition.

1 INTRODUCTION

The heat equation is an important parabolic partial differential equation (PDE) which refer to the distribution of heat (or variation in temperature) in a given region over time. For better understanding of this paper, it is very significant that we realize the difference between heat and temperature. Heat is a procedure of energy transfer as a result of temperature difference between the two points. Thus, 'heat' is used to describe the energy transported through the heating process. On the other hand, temperature is a physical property of matter that describes the warmness or coldness of an object or environment. Therefore, no heat would be replaced between bodies of the same temperature. The heat equation is of vital importance in diverse scientific fields. In mathematics, this equation is the prototypical parabolic partial differential equation. In probability theory, the heat equation is coupled with the study of Brownian motion via the Fokker-Planck equation. In financial mathematics, this is use to solve the Black-Scholes partial differential equation. The diffusion equation, a general version of the heat equation, rises in joining with the study of chemical diffusion and other related processes. Many researchers have already been worked on it. Abbas, Z., Sajid, M., Hayat, T. 2006 [1] MHD boundary-layer flow of an upper-convected Maxwell fluid in a porous channel" Unsworth, J., Duarte, F. J. 1979 [2] Heat diffusion in a solid sphere and Fourier Theory. Borjini, M.N., Mbow, C., Daguenet, M. 1999 [3] Numerical analysis of combined radiation and unsteady natural convection within a horizontal annular space. William F. Ames. [4] Numerical Methods for Partial Differential Equations. F. Durbin [5] Numerical inversion of Laplace transforms: an efficient improvement to Dubner and Abate's method. M. Dehghan [6] Numerical schemes for one-dimensional parabolic equations with nonstandard initial Condition. Randall J. Leveque [7] Numerical methods for conservation laws. Nicholas J. Higham [8] Accuracy and stability of Numerical Algorithms. Young-San Park, Jong-Jin Baik [9] Analytical solution of the heat equation for a ground level finite area source.

The research be made up of the following steps:

In section 2, presents a short discussion of heat equation. Based on the study of the general finite difference method We apply the finite difference scheme to obtain the numerical solution for the heat equation respectively as an IBVP. In section 3, we study the exact solution. the stability condition of the numerical scheme in order to avoid oscillation. In section 4, we present an algorithm for the numerical solution and we develop a computer programming code for the implementation of the numerical scheme. In section 5, we implement the numerical scheme to estimate the accuracy and efficiency test of heat equation for

various thermal conductivity. In fine, the conclusions of the work are given in the last segment.

2 GOVERNING EQUATION

Consider a homogenous, insulated bar or a wire of length L. Let the bar be located on the x axis on the interval $[0,l]$. Let the rod have a supply of heat. Let $u(x,t)$ denote the temperature in the rod at any instant time t. The problem is to study the flow of warmth within the rod. The PDE governs the flow of heat on the rod is given the equation.

$$\frac{\delta u}{\delta t} = \alpha \ \frac{\delta^2 u}{\delta x^2} \quad 0 \le x \le l, t \ge 0 \tag{1}$$

Where α is a constant and depends on the material properties of the rod. In order that the solution of the problem exists and is unique, we want to impose the subsequent conditions:

1) Initial condition, at time t=0, the temperature is prescribed,

$$u(x,0) = f(x), 0 \le x \le l$$

2) Boundary conditions: Since the bar is of length l, boundary condition at x=0 and at x=l are to be prescribed. These conditions are of the following types

   (a) Temperature at the ends of the bar prescribed.

$$u(0,t) = g(t), u(l,t) = h(t), t \ge 0$$

   (b) One end of the bar, say at x=0 & insulated. This implies the condition that,

$$\frac{\delta u}{\delta x} = 0 \text{ at } x = 0$$

2.1 Exact solution of heat equation

Fourier transform

$$u(k,t) = \int_{-\infty}^{\infty} u(x,t) e^{ixk} dx \tag{2}$$

Inverse Fourier transform

$$u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} u(k,t) e^{ixk} dx \tag{3}$$

$$\frac{\partial u}{\partial t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\partial u}{\partial t} e^{ixk} dk$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} u(k,t)(ik)^2 e^{ixk} dk$$

Now Heat equation we can write,

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

$$\Rightarrow \frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial x^2} = 0$$

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\partial u}{\partial t} e^{ixk} dk - \alpha \frac{1}{2\pi} \int_{-\infty}^{\infty} u(k,t) i^2 k^2 e^{ixk} dk = 0$$

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{\partial u}{\partial t} + \alpha k^2 u(k,t) \right] e^{ixk} dk = 0$$

$$\therefore \frac{\partial u}{\partial t} = -\alpha k^2 u \qquad \Rightarrow \frac{\partial u}{u} = -\alpha k^2 \partial t$$

$$\Rightarrow \ln u = -\alpha k^2 t + \ln A$$

$$\Rightarrow u(k,t) = A e^{-\alpha k^2 t} \tag{4}$$

$$\therefore u(k,0) = A e^0 \qquad \Rightarrow u(k,0) = A$$

Let, $u(k,0) = 1$

$\therefore A = 1$ Since $u(k,0) = 1$

Then From Equation (4) we get,

$$u(k,t) = e^{-\alpha k^2 t}$$

Putting the value of $u(k,t) = e^{-\alpha k^2 t}$ in equation (3) we get

$$u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\alpha k^2 t} e^{ixk} dk$$

$$\Rightarrow u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\alpha t} \left(k^2 - \frac{ix}{\alpha t} k \right) dk$$

$$\Rightarrow u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\alpha t} \left[\left(k - \frac{ix}{\alpha t} \right)^2 + \frac{x^2}{4\alpha^2 t^2} \right] dk$$

$$\Rightarrow u(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-\alpha t}\frac{x^2}{4\alpha^2t^2}e^{-\alpha t}\left(k-\frac{ix}{2\alpha t}\right)^2dk$$

Let $k-\frac{ix}{2\alpha t}=y$

$\Rightarrow dk=dy$

$$\Rightarrow u(x,t)=\frac{1}{2\pi}e^{\frac{-x^2}{4\alpha t}}\int_{-\infty}^{\infty}e^{-\alpha ty^2}dy$$

Let
$$\alpha ty^2=z^2$$
$$\Rightarrow z=y\sqrt{\alpha t}$$
$$\Rightarrow dz=dy\sqrt{\alpha t}$$
$$\Rightarrow dy=\frac{1}{\sqrt{\alpha t}}dz$$

$$\Rightarrow u(x,t)=\frac{1}{2\pi}e^{\frac{-x^2}{4\alpha t}}\int_{-\infty}^{\infty}e^{-z^2}\frac{1}{\sqrt{\alpha t}}dz$$

$$\Rightarrow u(x,t)=\frac{1}{2\pi}e^{-\frac{x^2}{4\alpha t}}.\frac{1}{\sqrt{\alpha t}}.\int_{-\infty}^{\infty}e^{-z^2}dz$$

$$\Rightarrow u(x,t)=\frac{1}{2\pi}e^{\frac{-x^2}{4\alpha t}}.\frac{1}{\sqrt{\alpha t}}.\sqrt{\pi}$$

$$\therefore u(x,t)=\frac{1}{\sqrt{4\pi\alpha t}}.e^{\frac{-x^2}{4\alpha t}}$$

Which is the fundamental solution of the one dimensional heat equation.

3 NUMERICAL METHODS FOR GOVERNING EQUATION

We consider the specific one dimensional heat equation as an initial and boundary condition

$$\frac{\partial u}{\partial t}=\alpha\frac{\partial^2u}{\partial x^2}\quad \text{for } 0\le x\le l, t>0$$

With I.C $u(x,0)=f(x)$ for $0\le x\le l, t=0$

B.C $u(0,t)=g(t), u(l,t)=h(t), t>0$

Finite difference techniques for solving the one dimensional heat equation can be considered according to the number of spatial grid points involved, the number of time levels used, whether they are explicit or implicit in nature.

3.1 Explicit centered difference scheme

$$\frac{\partial u}{\partial t}=\alpha\frac{\partial^2u}{\partial x^2}\qquad (5)$$

The simplest numerical discretization of heat (5) is the explicit centered difference scheme which is obtained by a first order forward difference in time and the second order centered difference in space.

Forward difference in time: $\frac{\partial u}{\partial t}(x_i^k)\approx\frac{u_i^{k+1}-u_i^k}{\tau}$

Centered difference in space:

$$\frac{\partial^2u}{\partial x^2}(x_i^k)\approx D^+D^-x_i^k=\frac{1}{h^2}[u_i^{k+1}-2u_i^k+u_{i-1}^k]$$

Therefore, the explicit centered difference scheme of the heat equation (5) is

$$\frac{u_i^{k+1}-u_i^k}{\tau}-\frac{\alpha}{h^2}[u_i^{k+1}-2u_i^k+u_{i-1}^k]=0$$

$\Rightarrow u_i^{k+1}=u_i^k+\frac{\alpha\tau}{h^2}[u_i^{k+1}-2u_i^k+u_{i-1}^k]$ Where,

$$\Rightarrow u_i^{k+1}=u_i^k+\gamma[u_i^{k+1}-2u_i^k+u_{i-1}^k]+$$

$$\therefore u_i^{k+1}=(1-2\gamma)u_i^k+\gamma[u_i^{k+1}+u_{i-1}^k]$$

$$\gamma=\frac{\alpha\tau}{h^2}$$

3.2 Crank Nicolson Scheme

In order to achieve the second order accuracy in both space and time, we can discretize as

$$\frac{u_i^{k+1}-u_i^k}{\tau}=\frac{\alpha}{2}\left[\frac{u_{i+1}^{k+1}-2u_i^{k+1}+u_{i-1}^{k+1}}{h^2}\right]+\frac{\alpha}{2}\left[\frac{u_{i+1}^k-2u_i^k+u_{i-1}^k}{h^2}\right]$$

$$\Rightarrow u_i^{k+1}-u_i^k=\frac{\alpha\tau}{2h^2}\left[u_{i+1}^{k+1}-2u_i^{k+1}+u_{i-1}^{k+1}\right]+\frac{\alpha\tau}{2h^2}\left[u_{i+1}^k-2u_i^k+u_{i-1}^k\right.$$

$$\Rightarrow u_i^{k+1}+\frac{\gamma}{2}\left[2u_i^{k+1}-u_{i+1}^{k+1}-u_{i-1}^{k+1}\right]=u_i^k-\frac{\gamma}{2}\left[2u_i^k-u_{i+1}^k-u_{i-1}^k\right]$$

For $i=1$

$$u_i^{k+1}+\frac{\gamma}{2}\left[2u_1^{k+1}-u_2^{k+1}-u_0^{k+1}\right]=u_1^k-\frac{\gamma}{2}\left[2u_1^k-u_2^k-u_0^k\right]$$

$$\Rightarrow u_i^{k+1}+\frac{\gamma}{2}\left[2u_1^{k+1}-u_2^{k+1}\right]=u_1^k-\frac{\gamma}{2}\left[2u_1^k-u_2^k\right]+\frac{\gamma}{2}\left[u_0^{k+1}+u_0^k\right]=u_1^k-\frac{\gamma}{2}\left[2u_1^k-u_2^k\right]$$

For $i=2$

$$u_2^{k+1}+\frac{\gamma}{2}\left[2u_2^{k+1}-u_3^{k+1}-u_1^{k=1}\right]=u_2^k-\frac{\gamma}{2}\left[-u_1^k+2u_2^k-u_1^k\right]$$

$$\Rightarrow u_2^{k+1}+\frac{\gamma}{2}\left[-u_1^{k+1}+2u_2^{k+1}-u_3^{k+1}\right]=u_2^k-\frac{\gamma}{2}\left[-u_1^k+2u_2^k-u_3^k\right]$$

And so on.

In matrix vector form that is

$$\left(I+A\frac{\gamma}{2}\right)\underline{u^{k+1}}=\left(I-A\frac{\gamma}{2}\right)\underline{u^k}$$

Where

$$A=\begin{bmatrix} 2 & -1 & 0 & 0 & . & . & . & . & . & . & 0 \\ -1 & 2 & -1 & 0 & . & . & . & . & . & . & . \\ 0 & -1 & 2 & -1 & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ 0. & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & -1 & 2 & -1 \\ 0 & . & . & . & . & . & . & . & 0 & -1 & 2 \end{bmatrix}$$

3.3 Stability condition for explicit Centered difference scheme

If $0<\gamma<\frac{1}{2}$ then the solution at new time is a weighted average of the solution at old time .This implies a discrete maximum principle $\max|u_i^{k+1}|\le\max|u_i^k|$ and therefore numerically stability. The Explicit Centered Difference Scheme is stable if $\gamma<\frac{1}{2}$ that is $\frac{\alpha\tau}{h^2}<\frac{1}{2}$.

3.4 Von Neumann Stability condition for CNS scheme

Consider one dimensional heat equation is

$$\frac{\partial u}{\partial t}=\alpha\frac{\partial^2 u}{\partial x^2}, x\in(a,b), t\in(0,T) \qquad (6)$$

With initial condition, $u(0,x)=u_0(x)$

And boundary condition

And $\begin{matrix} u(t,a)=u_a(t); x\in(a,b) \\ u(t,b)=u_b(t); T\in(0,T) \end{matrix}$

In order to obtain Crank-Nicolson schema, we discretize $\frac{\partial u}{\partial t}$ and $\frac{\partial^2 u}{\partial x^2}$ at any discrete point (t^n, x_j) as follow

$$\frac{\partial u}{\partial t}\approx\frac{u_j^{n+1}-u_j^n}{\Delta t}$$ (2nd order central difference formula)

For $(t)th$ time step

$$\frac{\partial^2 u}{\partial x^2}\approx\frac{u_{j-1}^n-2u_j^n+u_{j+1}^n}{(\Delta x)^2}$$ (2nd order central difference formula)

For $(t+k)th$ time step

$$\frac{\partial^2 u}{\partial x^2}\approx\frac{u_{j-1}^{n+1}-2u_j^{n+1}+u_{j+1}^{n+1}}{(\Delta x)^2}$$ (2nd order difference formula)

Using these into (6) the simplest numerical discretization is

$$\frac{u_j^{n+1}-u_j^n}{\Delta t}=\frac{\alpha}{2}\left(\frac{u_{j-1}^n-2u_j^n+u_{j+1}}{(\Delta x)^2}+\frac{u_{j-1}^n-2u_j^n+u_{j+1}^n}{(\Delta x)^2}\right)$$

$$\Rightarrow u_j^{n+1}=u_j^n+\frac{\alpha\Delta t}{2(\Delta x)^2}\left(u_{j-1}^{n+1}-2u_j^{n+1}+u_{j+1}^{n+1}+u_{j-1}^n-2u_j^n+u_{j+1}^n\right)$$

$$\Rightarrow u_j^{n+1}=u_j^n+\frac{\gamma}{2}\left(u_{j-1}^{n+1}-2u_j^{n+1}+u_{j+1}^{n+1}+u_{j-1}^n-2u_j^n+u_{j+1}^n\right) \qquad (7)$$

Let $u_j^n=\xi^n e^{ik/\Delta x}$ into (7) we get,

$$\xi^{n+1}e^{ik/\Delta x} = \xi^n e^{ik/\Delta x} + \frac{\gamma}{2}\begin{pmatrix} \xi^{n+1}e^{ik(j-1)\Delta x} - 2\xi^{n+1}e^{ikj\Delta x} + \\ \xi^{n+1}e^{ik(j+1)\Delta x} + \xi^n e^{ik(j-1)\Delta x} \\ -2\xi^n e^{\frac{ik}{\Delta x}} + \xi^n e^{ik(j+1)\Delta x} \end{pmatrix}$$

$$\Rightarrow \xi^{n+1} = \xi^n + \frac{\gamma}{2}\left(e^{-ik\Delta x} - 2 + e^{ik\Delta x}\right)\xi^{n+1} + \frac{\gamma}{2}\left(e^{-ik\Delta x} - 2 + e^{ik\Delta x}\right)\xi^n$$

$$\Rightarrow \xi^{n+1} = \xi^n + \frac{\gamma}{2}\left(2\cos k\Delta x - 2\right)\xi^{n+1} + \frac{\gamma}{2}\left(2\cos k\Delta x - 2\right)\xi^n$$

$$\Rightarrow \xi^{n+1} = \xi^n - \gamma\left(1 - \cos k\Delta x\right)\xi^{n+1} - \gamma\left(1 - \cos k\Delta x\right)\xi^n$$

$$\Rightarrow \xi^{n+1} = \xi^n - 2\gamma\sin^2\left(\frac{k\Delta x}{2}\right)\xi^{n+1} - 2\gamma\sin^2\left(\frac{k\Delta x}{2}\right)\xi^n$$

$$\Rightarrow \left(1 + 2\gamma\sin^2\left(\frac{k\Delta x}{2}\right)\right)\xi^{n+1} = \left(1 - 2\gamma\sin^2\left(\frac{k\Delta x}{2}\right)\right)\xi^n$$

$$\Rightarrow \xi^{n+1} = \frac{1 - 2\gamma\sin^2\left(\frac{k\Delta x}{2}\right)}{1 + 2\gamma\sin^2\left(\frac{k\Delta x}{2}\right)}\xi^n$$

Therefore, we find the amplification factor

$$\xi = \frac{1 - 2\gamma\sin^2\left(\frac{k\Delta x}{2}\right)}{1 + 2\gamma\sin^2\left(\frac{k\Delta x}{2}\right)}$$

Since $\sin^2\left(\frac{k\Delta x}{2}\right) \geq 0$ and $\gamma \rhd 0$, it follows that $|\xi| \leq 1$

Consequently, the Crank-Nicolson method is unconditionally stable.

4 ALGORITHM FOR THE NUMERICAL SOLUTION

To find the numerical solution of the model, we have to accumulate some variables which are offered in the following algorithm.

Input: nx and nt are the number of spatial and temporal mesh points respectively.

t_f, the right end of $(0, T)$

x_d, the right end point of $(0, b)$

u_0, the initial concentration density, apply as initial condition

u_a, left hand boundary condition

α, heat diffusion rate

Output: $u(x,t)$ the solution matrix

Initialization: $dt = \frac{T-0}{nt}$ the temporal grid size

$dx = \frac{b-0}{nx}$ the spatial grid size

$\gamma = \frac{\alpha\tau}{h^2}$

Algorithm of ECDS

Step 1. Calculation for concentration profile of explicit centered difference scheme

$for\ k = 1$ to nt
$for\ i = 2$ to nx
$u_i^{k+1} = (1-2\gamma)u_i^k + \gamma[u_i^{k+1} + u_{i-1}^k]$
end
end

Step 2: output $u(x,t)$

Step 3: Figure Presentation

Step 4: Stop

Algorithm of CNS

A =left hand matrix of the scheme

B =right hand matrix of the scheme

Step 1. Calculation for concentration profile of Crank-Nicolson scheme

$for\ k = 2$ to $nt + 1$
$uu = u(2:nx, k-1)$
$C = B * uu$
$u(2:nx, k) = A \backslash C$
end

Step 2: output $u(x,t)$

Step 3: Figure Presentation

Step 4: Stop

5 COMPUTATIONAL RESULTS AND DISCUSSION

We implement two numerical finite difference schemes that are first order Explicit Centered Difference Scheme (EUDS) and second order Crank-Nicolson Scheme (CNS) by computer programming code and perform numerical simulation as described below.

5.1 Comparative study of heat equation in Different Time Step

We present numerical simulation results based on first order i.e. explicit centered difference scheme (EUDS) and second order Crank-Nicolson scheme (CNS). **Figure-1** shows temperature distribution of exact solution in different time step. **Figure-2** shows Comparison of temperature distribution of exact, ECDS and CNS solution in different time step α =0.25 $m^2 s^{-1}$. From the following figure we see that the temperature distribution of CNS and ECDS are close nearer to exact solution. In **figure-3** shows Comparison of temperature distribution of exact, ECDS and CNS solution in different time step at $\Delta t = 0.007$ and $\Delta x = 0.0287$ where solid red line represents the exact solution, the dot(blue) line represents the CNS solution and the solid green line represents the ECDS solution. In **figure-4** Temperature distribution of exact, ECDS and CNS solution for last time step at $\Delta t = 0.067$ and $\Delta x = 0.03$. **In figure-5** Temperature distribution of exact, ECDS and CNS solution for last time step at $\Delta t = 0.007$ and $\Delta x = 0.0287$.

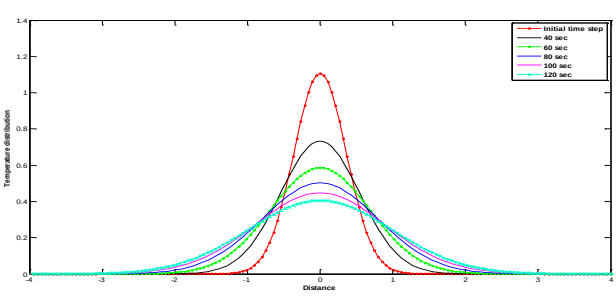

Figure-1: Temperature distribution of exact solution in different time step

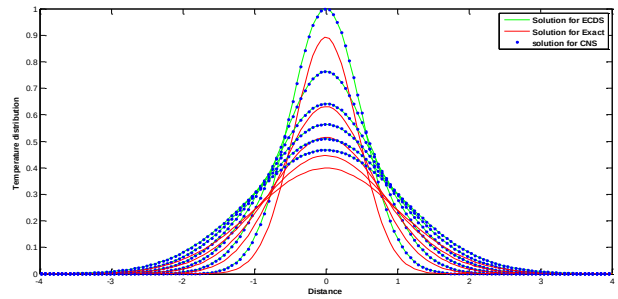

Figure-2: Comparison of temperature distribution of exact, ECDS and CNS solution in different time step α =0.25 $m^2 s^{-1}$

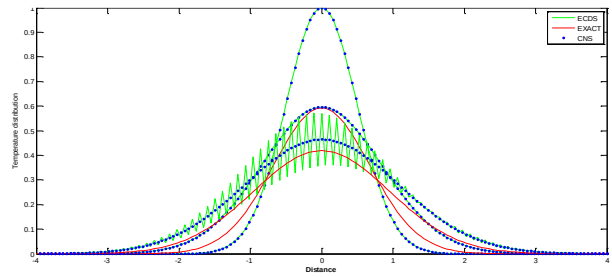

Figure-3: Comparison of temperature distribution of exact, ECDS and CNS solution in different time step at $\Delta t = 0.007$ and $\Delta x = 0.0287$

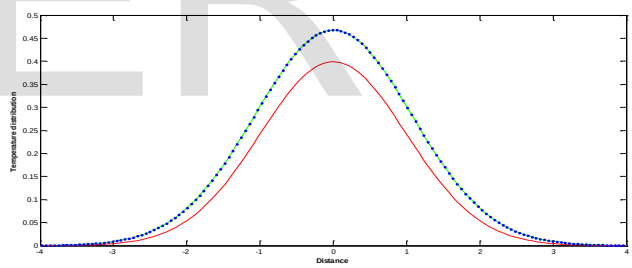

Figure-4: Temperature distribution of exact, ECDS and CNS solution for last time step at $\Delta t = 0.067$ and $\Delta x = 0.03$

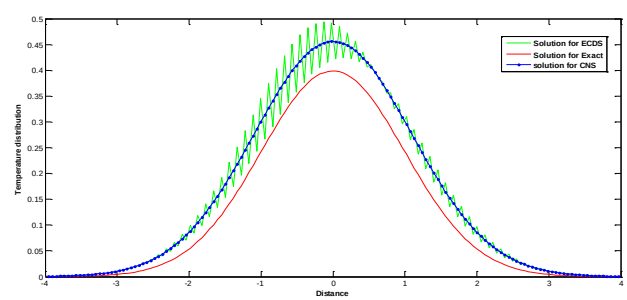

Figure-5: Temperature distribution of exact, ECDS and CNS solution for last time step at $\Delta t = 0.007$ and $\Delta x = 0.0287$

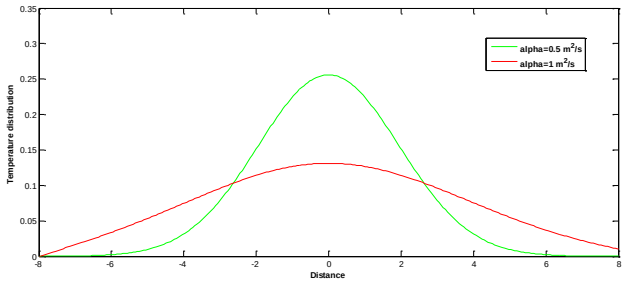

Figure-6: Temperature distribution of CNS solution for

α =0.5 $m^2 s^{-1}$ and α =1 $m^2 s^{-1}$.

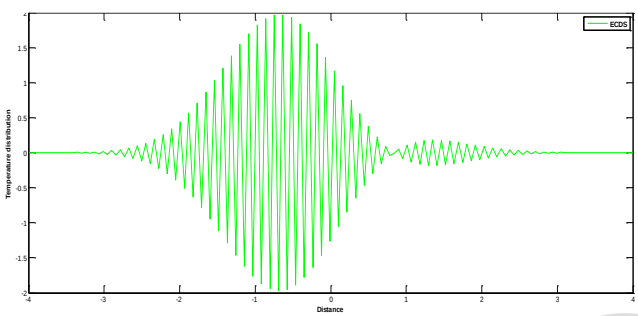

Figure-7: Temperature distribution of CNS for $\alpha = 0.3\, m^2 / s$, $\Delta t = 0.01$ and $\Delta x = 0.03$.

From **figure-6** shows the numerical solution of Crank-Nicolson scheme for $\alpha = 0.5 m^2 s^{-1}$ and $\alpha = 1\, m^2 s^{-1}$ where has no unstable condition(no more zigzag) but in **figure-7** shows the solution of ECDS for $\alpha = 0.3\, m^2 s^{-1}$ where has enough zigzag and it demonstrations that CNS gives better pointwise solution then explicit centered difference scheme.

CONCLUSION

The study has presented the exact and numerical solution of heat equation by using first order explicit centered difference scheme and 2nd order Crank-Nicolson scheme with an initial condition and two boundary condition. Here numerical experiment is presented graphically for different thermal conductivity. We have shown that the numerical scheme of ECDS is conditionally stable and CNS is unconditionally stable and studying the stability condition in terms of time step, it's observed that CNS is more efficient and accurate then explicit centered difference scheme. The results show that the temperature distribution is moving with varying the heat diffusive coefficient with respect to time and space.

REFERENCES

[1] Abbas, Z., Sajid, M., Hayat, T. [2006], "MHD boundary-layer flow of an upper-convicted Maxwell fluid in a porous channel". Theoretical and Computational Fluid Dynamics, Vol. 20 pp.29-38.

[2] Unsworth, J.; Duarte, F. J. (1979), "Heat diffusion in a solid sphere and Fourier Theory".

[3] Borjini, M.N., Mbow, C., Daguenet, M. [1999], "*Numerical analysis of combined radiation and unsteady natural convection within a horizontal annular space*", International Journal of Numerical Methods for Heat & Fluid Flow, vol.231pp. 34-39.

[4] F. Durbin, "Numerical inversion of Fourier transforms: An Efficient improvement to Dubner and Abate's method," The Computer Journal, vol. 17, no. 4, pp. 371–376, 1974.

[5] M. Dehghan, "Numerical schemes for one-dimensional parabolic equations with nonstandard initial condition," Applied Mathematics and Computation, vol. 147, no. 2, pp. 321–331, 2004.

[6] Randall J. Leveque, "Numerical methods for conservation laws," second edition, 1992, springer.

[7] J.Higham, "Stability of Numerical Solution," Society of Industrial and Applied Mathematics, Philadelphia, 2005.

[8] Young-San Park, Jong-Jin Baik, "Analytical solution of the heat equation for a ground level finite area source," Atmospheric Environment 42, 9063-9069, 2008.

[9] Al-Odat, M.A. Al-Nimr, M. Hamdan, [2003]. Thermal stability of composite superconducting tape under the effect of a two- dimensional dual-phase-lag heat conduction model journal of mass and heat transfer.

[10] Nicholas J.Higham, "Accuracy and stability of numerical Algorithms", Society of Industrial and Applied Mathematics, Philadelphia, 1996.

[11] Dilip Kumar Jaiswal, Atul Kumar, Raja Ram Yadav, "*Analytical solution of the heat equation with temporally dependent Coefficients*", Journal of water Resource and Protection, 2011, 3, 76-84.

[12] S. B.Yuste and L.Acedo, "*An explicit finite difference method and a new Von-Neumann type stability analysis for fractional diffusion equations*", Vol. 42, No.5, pp.1862-1874, Society for Industrial and Applied Mathematics 2005.

Most. Rozina Khatun
Lecturer, Department of Mathematics
Bangabandhu Sheikh Mujibur Rahaman Science and Technology University , Gopalgonj-8100
Email:rozina.rekha24@gmail.com

Md. Shajib Ali
Assistent Professor, Dept of Mathematics
Islamic University, Kushtia, Bangladesh.
Email: shajib_301@yahoo.co.in